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Simplified Unsteady Aerodynamic Concepts with Application to Parameter Estimation

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A simplified aerodynamic force model based on Prandtl's lifting line theory and trailing vortex concepts has been developed to account for unsteadiness in the aircraft dynamics. The wake is assumed to be compressed to a single shed vortex element of appropriate strength moving downstream at a speed sufficient to approximate the Wagner function. Results are presented illustrating the ability of the simplified theory to duplicate exact solutions in unsteady aerodynamics. Further, consideration is given to the utility of the model in a parameter identification application. A numerical example is given utilizing a least-squares computational algorithm to demonstrate the importance of including unsteady effects in the estimate of the aircraft parameters.

Nomenclature

A	= aspect ratio = b^2/S
b	= wing span
$\bar{c}$	= average chord
c	= chord
C_{L_α}	= lift curve slope = $\partial C_L/\partial\alpha$
C_m	= pitching moment coefficient = $\dfrac{\text{pitching moment}}{\frac{1}{2}\rho U^2 S\bar{c}}$
C_{m_q}	= $\dfrac{\partial C_m}{\partial(q\bar{c}/2U)}$
$C_{m_{\delta_e}}$	= $\partial C_m/\partial\delta_e$
C_x	= axial force coefficient = $\dfrac{\text{axial force}}{\frac{1}{2}\rho U^2 S}$
C_z	= normal force coefficient = $\dfrac{\text{normal force}}{\frac{1}{2}\rho U^2 S}$
$C_{z_{\delta_e}}$	= $\partial C_z/\partial\delta_e$
I_y	= moment of inertia about pitch axis
K,a,b,f,g,h	= constants
l_t	= distance from wing quarter-chord line to horizontal tail quarter-chord line
$\Delta\ell$	= increment in lift of two-dimensional wing due to unit step increase in angle of attack
ΔC_L	= change in lift coefficient of a three-dimensional wing due to a unit step increase in angle of attack
Δc_ℓ	= change in lift coefficient of a two-dimensional wing due to a unit step increase in angle of attack
q	= pitch rate
S	= reference area
s	= Laplace transform variable
$\bar{t}$	= nondimensional time = $Ut/(\bar{c}/2)$
t	= time
U	= freestream velocity
u	= perturbation in forward velocity
w	= downwash velocity at the ¾ chord of the wing, due to vortices
x_0	= distance of starting shed vortex downstream from wing ¾ chord line
α	= geometric angle of attack
$\dot{\bar{\alpha}}$	= $d\alpha/d\bar{t}$
Γ	= change in circulation due to a unit step increase in angle of attack
δ_e	= elevator deflection
ϵ	= downwash angle = w/U
$\Delta\epsilon$	= increment in downwash angle due to unit step increase in angle of attack
θ	= attitude angle
ω	= frequency
ρ	= air density

Subscripts

0	= initial condition
∞	= steady-state condition ($t=\infty$)
w	= wing
t	= horizontal tail
$(\dot{\ })$	= d/dt

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I. Introduction

SEVERAL references have shown that aerodynamic parameters and their variances, as determined from flight data, are influenced by the type of control input used to excite the aircraft motion.[1,2] These differences have generally been attributed to insufficient excitation of the aircraft states. However, another possibility is that unsteady aerodynamic effects, which are generally neglected in parameter estimation, might account for the observed differences in variances. The purpose of the present study was to develop simple concepts which would permit modeling of unsteady aerodynamic effects into parameter-extraction programs.

II. Simplified Model

The recent direction of unsteady lifting surface theory has been toward digital computer application to the classical formulations.[3-6] A vortex-lattice method of determining the transient lift buildup on a finite wing has been presented by Belotserkovskii.[3] Morino[4] has developed a numerical procedure to solve for the unsteady lift on a wing based on a Green's function formulation. Numerical implementation of the theoretical development of Ref. 4 is given in Ref. 5. A recent contribution to the field of unsteady aerodynamic theory with application to wing flutter is due to Edwards.[6] The referenced methods provide physically realistic mathematical representations of the aerodynamic loads. However, a real need exists for a simpler, but accurate, mathematical model for the loads for utilization in parameter estimation studies even though some physical realism may be sacrificed. The simplifying approximations used in the present paper are best illustrated relative to the Wagner function for two- and three-dimensional wings and corresponding downwash.

The Wagner Function

The approximation to the Wagner function

$$\frac{\Delta c_\ell}{2\pi} = 1 - \left(2 + \frac{1}{2}\,\frac{Ut}{c/2}\right)^{-1} \tag{1}$$

given by Garrick[7] can be thought of as being generated by a single bound vortex of strength Γ located at the wing quarter chord and a free vortex of strength $-\Gamma$ which is convected downstream with speed $U/2$ (Fig. 1). The value of the constant x_0 is determined by forcing the startup lift to match that computed by Jones.[8]

A similar concept was used to compute the lift response to a unit step change in angle of attack of a straight three-dimensional wing. In this case, the vortex system consists of a lifting line, two trailing vortices, and a shed vortex convected downstream at speed $U/2$ (Fig. 2). If the Biot-Savart equation is used to compute the downwash at the ¾ chord and the Kutta-Joukowski equation is used to relate lift to circulation, the change in lift is expressed as[9]

$$\frac{\Delta C_L}{(C_{L_\alpha})_\infty} = (1+\sqrt{A^2+1})\left\{\sqrt{A^2+1} + \left(1+A^2\left[\frac{x_0}{\bar{c}/2} + \frac{1}{2}\,\frac{Ut}{\bar{c}/2}\right]^{-2}\right)^{-1/2}\right\}^{-1} \tag{2}$$

Results from Eq. (2) are compared with Jones' results for elliptic wings of aspect ratios 3 and 6 (Fig. 3). The results are in excellent agreement, indicating that Eq. (2) is accurate.

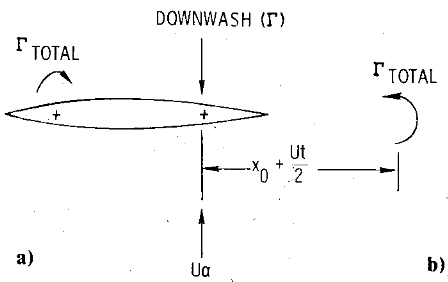

Fig. 1 Vortex representation of a two-dimensional wing: a) instant after α increases, b) system used in this study.

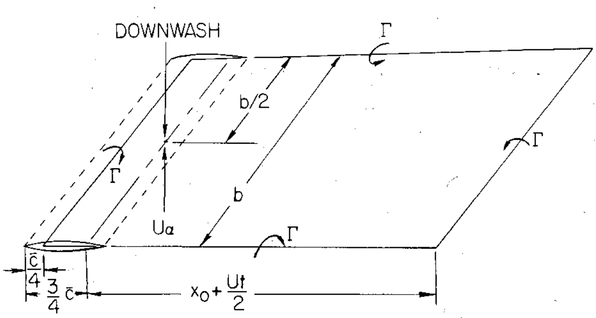

Fig. 2 Simplified vortex system for a three-dimensional unswept wing.

Downwash Behind a Wing

In determining the lift of a horizontal tail, it is necessary to compute the downwash at the tail caused by the wing lift. In a physical system, a vortex shed from the wing moves downstream at freestream velocity. The downwash created by the vortex as it crosses a given point is a discontinuous function since it changes sign abruptly. The downwash discontinuity reaches the tail at a time approximately equal to the tail length divided by the freestream velocity. In order to have the simplified model as consistent as possible with this physical fact, it was decided to assume the vortex system stretches downstream with freestream velocity, for the purpose of computing downwash at the horizontal tail. The downwash at a distance l_t behind the lifting line of the vortex and equidistant from the trailing vortices of the system shown in Fig. 4 is given by

$$w = \frac{\Gamma}{2\pi b}\left[\frac{1}{l_t}\sqrt{b^2+4l_t^2} + \frac{\sqrt{b^2+4(\bar{c}/2+x_0+Ut-l_t)^2}}{\bar{c}/2+x_0+Ut-l_t}\right] \tag{3}$$

Using the relationship $\epsilon = w/U$, and Eq. (2), it can be shown that for unit step in angle of attack

$$\frac{\partial\epsilon}{\partial\alpha} = \frac{1}{2\pi A}(C_{L_\alpha})_\infty (1+\sqrt{A^2+1})\Big/\left[\sqrt{A^2+1} + \sqrt{1+A^2\left(\frac{x_0}{\bar{c}/2}+\frac{Ut}{\bar{c}/2}\right)^{-2}}\right] \times\left[\sqrt{\left(\frac{b}{l_t}\right)^2+4} + \frac{\sqrt{b^2+4(\bar{c}/2+x_0+Ut-l_t)^2}}{\bar{c}/2+x_0+Ut-l_t}\right] \tag{4}$$

Equation (4) is limited in accuracy because of the simple vortex-system model, and can be shown to predict only about one-half of the value of $\partial\epsilon/\partial\alpha$ far behind elliptic wings under steady-state conditions (α constant, t very large). A more accurate version of Eq. (4) can be computed by first obtaining the steady-state value

$$\left(\frac{\partial\epsilon}{\partial\alpha}\right)_\infty = \frac{1}{2\pi A}(C_{L_\alpha})_\infty\left[2+\sqrt{\left(\frac{b}{l_t}\right)^2+4}\right] \tag{5}$$

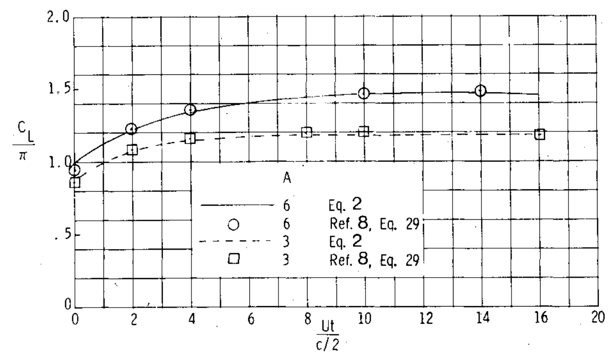

Fig. 3 Indicial lift for elliptic wings.

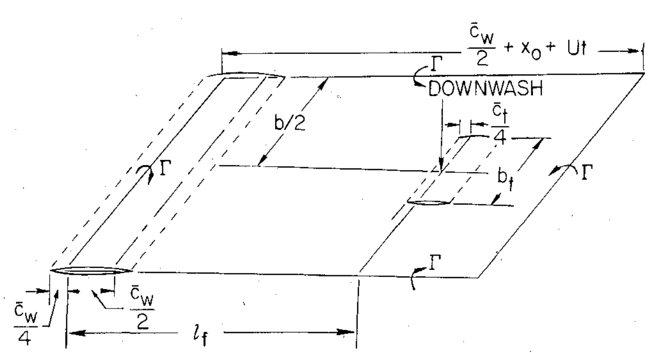

Fig. 4 Geometry of simplified vortex system for determination of downwash at horizontal tail.

Solving Eq. (5) for $(C_{L_\alpha})_\infty/2\pi A$ and substituting into Eq. (4) results in, for a unit step input in α,

$$\Delta\epsilon=\left(\frac{\partial\epsilon}{\partial\alpha}\right)_\infty\left[2+\sqrt{\left(\frac{A}{l_t/\bar{c}}\right)^2+4}\right]^{-1}\cdot\left\{(1+\sqrt{A^2+1})\Big/\left[\sqrt{A^2+1}+\sqrt{1+A^2\left(\frac{x_0}{\bar{c}/2}+\frac{Ut}{\bar{c}/2}\right)^{-2}}\right]\right\}$$
$$\times\left\{\sqrt{\left(\frac{A}{l_t/\bar{c}}\right)^2+4}+\sqrt{A^2+\left(1+\frac{x_0}{\bar{c}/2}+\frac{Ut}{\bar{c}/2}-\frac{l_t}{\bar{c}/2}\right)^2}\Big/\frac{1}{2}\left[1+\frac{x_0}{\bar{c}/2}+\frac{Ut}{\bar{c}/2}-\frac{l_t}{\bar{c}/2}\right]\right\} \quad (6)$$

The procedure for determining $\Delta\epsilon$ is to use the best source available for determining $(\partial\epsilon/\partial\alpha)_\infty$ at the desired downstream position (use, e.g., Ref. 10), and obtain the time dependence from the remaining terms of Eq. (6).

The effective angle of attack at the tail is equal to the geometric angle of attack minus the downwash angle, that is $(\alpha-\epsilon)$. The indicial lift of the horizontal tail (for unit change in effective angle of attack of the tail) is given by (assuming $x_0/(\bar{c}/2)\approx 1$)

$$\frac{\Delta C_{L_t}}{(C_{L_\alpha})_\infty}=(\sqrt{A_t^2+1})\Big/\left[\sqrt{A_t^2+1}\cdot\sqrt{1+A_t^2\left(\frac{x_0}{\bar{c}_t/2}+\frac{Ut}{\bar{c}_t/2}\right)^{-2}}\right] \quad (7)$$

III. Unsteady Effects for Arbitrary Angles of Attack

The lift and downwash associated with an isolated wing and the lift on a horizontal tail (behind a wing), due to an arbitrary angle of attack variation can be determined by use of Eqs. (2, 6, and 7) and Duhamel's integral formula according to

$$C_L=\int_0^t \Delta C_L(t-\tau)\dot{\alpha}(\tau)\mathrm{d}\tau \quad (8)$$

$$\epsilon=\int_0^t \Delta\epsilon(t-\tau)\dot{\alpha}(\tau)\mathrm{d}\tau \quad (9)$$

$$C_{L_t}=\int_0^t \Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau)-\dot{\epsilon}(\tau)]\mathrm{d}\tau \quad (10)$$

It is convenient to write Eqs. (2, 6, and 7) in terms of nondimensional distance traveled. The equations will be written in terms of wing half-chords traveled. Therefore, let

$$\bar{t}=\frac{Ut}{\bar{c}_w/2} \quad \text{and} \quad \bar{\alpha}=\frac{\mathrm{d}\alpha}{\mathrm{d}\bar{t}} \quad (11)$$

Equations (2, 6, and 7) can be written as (with $x_0/(\bar{c}/2)=1.0$)

$$\Delta C_L=(C_{L_\alpha})_\infty\left\{\frac{1+\sqrt{A^2+1}}{\sqrt{A^2+1}+\sqrt{1+A^2/[1+½\bar{t}]^2}}\right\} \quad (12)$$

$$\Delta\epsilon=\left(\frac{\partial\epsilon}{\partial\alpha}\right)_\infty\left[2+\sqrt{\left(\frac{A}{l_t/\bar{c}}\right)^2+4}\right]^{-1}\times\left[\frac{1+\sqrt{A^2+1}}{\sqrt{A^2+1}+\sqrt{1+A^2/(1+\bar{t})^2}}\right]$$
$$\times\left\{\sqrt{\left(\frac{A}{l_t/\bar{c}}\right)^2+4}+\sqrt{A^2+\left(2+\bar{t}-\frac{l_t}{\bar{c}/2}\right)^2}\Big/\frac{1}{2}\left(2+\bar{t}-\frac{l_t}{\bar{c}/2}\right)\right\} \quad (13)$$

$$\Delta C_{L_t}=(C_{L_\alpha})_{\infty,t}\left\{(1+\sqrt{A_t^2+1})\Big/\left[\sqrt{A_t^2+1}+\sqrt{1+\frac{A_t^2}{(1+\bar{t}\bar{c}_w/\bar{c}_t)^2}}\right]\right\} \quad (14)$$

IV. Numerical Checks of Basic Equations for Oscillating Wing-Tail Combination

The influence of wing oscillation on the lift of the horizontal tail can be found, through the use of Eqs. (13 and 14), by converting the equations by the Laplace transform. The conversion can be simplified by writing the equations in terms of expressions for which Laplace transforms are readily available. It was found that Eqs. (13 and 14) could be approximated very closely by equations of the form

$$\Delta C_{L_t}=(C_{L_\alpha})_{\infty,t}[1-ae^{-b\bar{t}(\bar{c}_w/\bar{c}_t)}] \quad (15)$$

$$\Delta\epsilon=\left(\frac{\partial\epsilon}{\partial\alpha}\right)_\infty\left[1-\frac{f}{l_t/\bar{c}_w-1-½\bar{t}}-ge^{-h\bar{t}}\right] \quad (16)$$

The constants a and b can be determined in closed form when the geometric characteristics of the wing and tail are specified. The constants f, g, and h can be determined by curve fitting. The purpose of going through this step is to obtain frequency-response curves to check against Jones and Fehlner[11] and thereby give increased confidence in Eq. (6).

The example given in Ref. 11 corresponds to $A_w=6$, $\bar{c}_w/\bar{c}_t=2.0$, $A_t=3$, $l_t/(\bar{c}_w/2)=7.29$. Reference 10 can be used to obtain, for these geometric parameters

$$(C_{L_\alpha})_{\infty,t}=3.363 \quad \text{and} \quad \left(\frac{\partial\epsilon}{\partial\alpha}\right)_\infty=0.50$$

For the given geometric parameters the constants involved in Eqs. (15) and (16) were found to be: $a=0.299$, $b=0.824$, $f=1.2402$, $g=0.617$, $h=0.0706$.

Making use of these constants and substituting Eqs. (15) and (16) into Eq. (10) and taking the Laplace transform

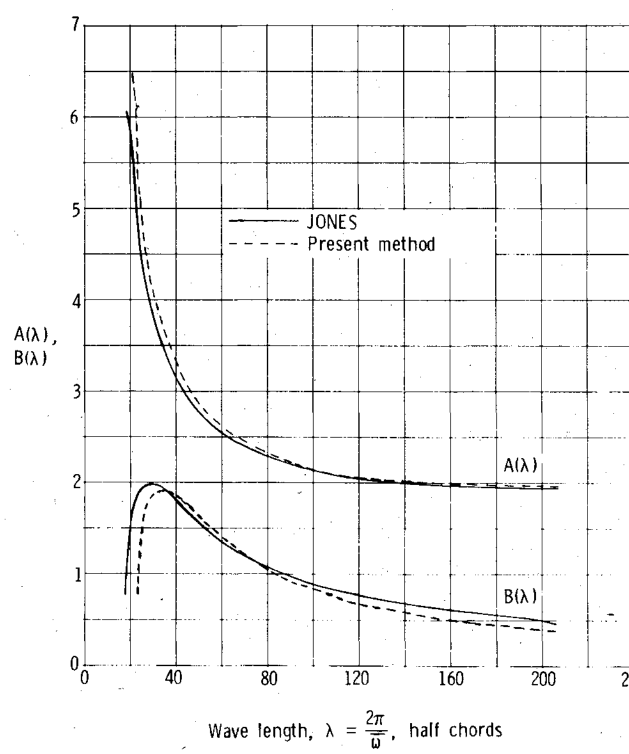

Fig. 5 Lift functions for horizontal tail of airplane in vertical oscillation without pitching, $C_{L_t}(i\omega)=A(\omega)+iB(\omega)$.

results in

$$C_{L_t}(s)=3.363\Big(1-\frac{0.299s}{s+0.824}\Big)\alpha(s)-1.682\Big(1-\frac{0.299s}{s+0.824}\Big)$$
$$\times\Big[1-2.480s\,e^{-5.29s}\mathrm{Ei}(5.29s)-\frac{0.617s}{s+0.0706}\Big]\alpha(s) \qquad (17)$$

where Ei is the exponential integral.[12]

The first term of Eq. (17) represents the lift of the isolated horizontal tail, and the second term accounts for the downwash behind the wing. When the wing-tail combination is performing a plunging oscillation, the influence of the oscillation on the lift of the trail can be determined by substituting $s=i\omega$ in Eq. (17). The real and imaginary parts of the results are shown in Fig. 5 and compared with results of Ref. 11. The comparison is quite good, which indicates that Eq. (6) adequately represents the downwash behind a wing following a step input in α (with no rotation).

V. Unsteady Longitudinal Equations

Equations (2) and (6) are basic in accounting for unsteady aerodynamic effects for longitudinal motion. These effects are introduced in the perturbated equations of motion, which can then be used for parameter estimation.

The assumed longitudinal equations of motion have the form

$$\dot{u}=-g\theta+(\rho U^2S/2m)C_x \qquad (18)$$

$$\dot{\alpha}=q+(\rho US/2m)C_z \qquad (19)$$

$$\dot{q}=(\rho U^2S\bar{c}/2I_y)C_m \qquad (20)$$

$$\dot{\theta}=q \qquad (21)$$

The axial force coefficient is expressed as

$$C_x=C_{x_\alpha}\alpha \qquad (22)$$

The effect of unsteadiness is introduced into the equations of motion through C_z and C_m according to the following

$$C_z=\int_0^t\Big\{\Delta C_{L_w}(t-\tau)\dot{\alpha}(\tau)$$
$$+(S_t/S_w)\Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau)-\dot{\epsilon}(\tau)]\Big\}\mathrm{d}\tau+C_{z_{\delta_e}}\delta_e \qquad (23)$$

$$C_m=-\frac{l_tS_t}{\bar{c}_wS_w}\int_0^t\Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau)-\dot{\epsilon}(\tau)]\mathrm{d}\tau$$
$$+C_{m_q}\frac{q\bar{c}_w}{2U}+C_{m_{\delta_e}}\delta_e \qquad (24)$$

Substituting Eqs. (22-24) into Eqs. (18-21) results in the final equations of motion

$$\dot{u}=-g\theta+(\rho U^2S/2m)C_{x_\alpha}\alpha \qquad (25)$$

$$\dot{\alpha}=q-\frac{\rho US}{2m}\Big\{\int_0^t\Delta C_{L_w}(t-\tau)\dot{\alpha}(\tau)\mathrm{d}\tau$$
$$+\frac{S_t}{S_w}\int_0^t\Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau)-\dot{\epsilon}(\tau)]\mathrm{d}\tau\Big\}+C_{z_{\delta_e}}\delta_e \qquad (26)$$

$$\dot{q}=\frac{-\rho U^2S\bar{c}_w}{2I_y}\Big\{\frac{l_tS_t}{\bar{c}_wS_w}\int_0^t\Delta C_{L_t}(t-\tau)[\dot{\alpha}(\tau)-\dot{\epsilon}(\tau)]\mathrm{d}\tau$$
$$-C_{m_q}\frac{qc_w}{2U}+C_{m_{\delta_e}}\delta_e\Big\} \qquad (27)$$

$$\theta=q \qquad (28)$$

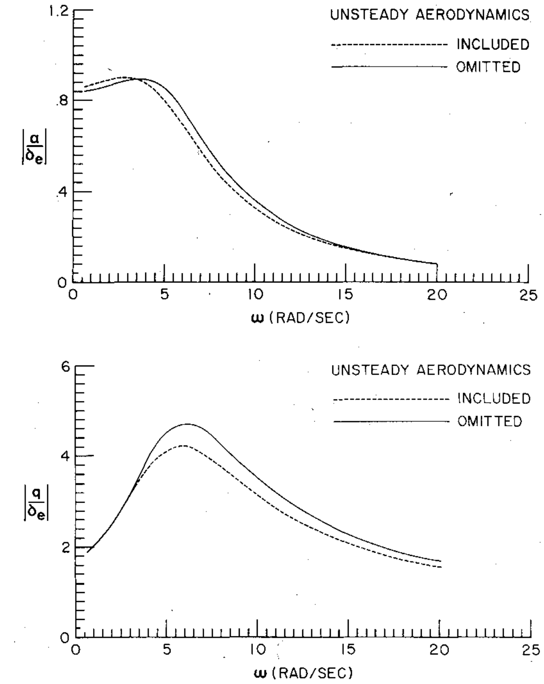

Fig. 6 Frequency response of Navion to unit impulse in elevator.

The integral equations represented by Eqs. (26) and (27) can be solved numerically or by Laplace transform techniques, provided the approximations for ΔC_L and $\Delta\epsilon$ discussed previously are employed.

VI. Parameter Estimation Results

To assess the effect of the exclusion of unsteady aerodynamics in the force and moment models used in contemporary estimation algorithms, a numerical example is considered. In this example, simulated flight data are generated using the unsteady aircraft dynamics as expressed by Eqs. (25-28). The lightweight Navion aircraft was chosen as subject since its characteristics have been studied extensively from flight test data.[13] A trimmed flight condition at 5000 ft altitude and speed of 240 ft/s was used. The frequency response of the aircraft using both unsteady and quasisteady models is shown in Fig. 6. The initial effect of unsteadiness is seen to be quite pronounced in the pitch rate response.

For the parameter-extraction process, a maximum likelihood algorithm was used. The algorithm was developed

Table 1 Parameter estimates using unsteady data

Parameter	True value	Estimated value	Standard deviation
$(C_{L_\alpha})_{\infty,t}$	3.88	3.35	0.005
C_{m_q}	−18.10	−20.19	0.033
$C_{z_{\delta_e}}$	−0.51	0.61	0.010
$C_{m_{\delta_e}}$	−1.42	−1.37	0.001

Table 2 Correlation matrix for the estimates of Table 1

Parameter	$(C_{L_\alpha})_{\infty,t}$	C_{m_q}	$C_{z_{\delta_e}}$	$C_{m_{\delta_e}}$
$(C_{L_\alpha})_{\infty,t}$	1	...	...	...
C_{m_q}	0.292	1	...	...
$C_{z_{\delta_e}}$	0.681	−0.059	1	...
$C_{m_{\delta_e}}$	0.205	0.817	0.039	1

on the basis of quasisteady aerodynamic theory; that is, no unsteadiness was included in the math model of the aerodynamic forces and moments used in the algorithm. A sampling rate of 40/rad was used in the data acquisition.

First, an estimation was performed using the algorithm with quasisteady simulated data for purposes of reference in the comparison. Next, an estimation was performed using the algorithm with unsteady simulated data. The results of these estimates are shown in Tables 1 and 2. A comparison of the actual with the estimated values indicates clearly the degradation in the estimates of the parameters due to the exclusion of unsteady effects in the estimation algorithm. The most significant difference appears to be in the estimate of the dynamic stability derivative C_{m_q}. This is not unexpected since the unsteadiness was seen to have a pronounced effect on the pitch rate response of the aircraft in Fig. 6.

VII. Concluding Remarks

A simplified aerodynamic force model based on lifting-line concepts has been developed to account for unsteady aerodynamics of a wing or wing-tail combination undergoing a change in angle of attack. Results from use of this model compare well with those of more complex methods.

A numerical example verified that the effect of unsteady aerodynamics is to cause additional uncertainty in the estimation of the aircraft parameters if the estimation algorithm is based only on quasisteady flow theory.

Acknowledgments

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